TECHNICAL AIDS

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A Control Chart for Parts-Per-Million Nonconforming Items

ONE alternative to monitoring the proportion nonconforming with a p chart when the occurrence of nonconforming items is in the parts-per-million (PPM) range is to use a chart of the number of items produced until a nonconforming item is found. This technique is not new (see Montgomery (1991)); but here we discuss the underlying assumptions and suggest a simple transformation that will allow the usual supplementary tests for special causes to be applied.

We will assume that "nonconforming" has a sound operational definition that is consistently applied. Further we will assume that the rate of occurrence of nonconforming items is adequately approximated by a Poisson distribution. It is known that the times between occurrences of Poisson-distributed events are independent and exponentially distributed. If a production process has a constant speed, then counting the number of items produced can serve as a surrogate for time. We shall count the numbers of conforming items produced between the occurrences of nonconforming items and refer to these as "inter-event counts".

In principle the problem is solved in that the distribution of what is to be measured (actually counted) is known and can be used to set up a control chart. But the exponential distribution is highly skewed and would give a chart for which no parametric supplementary rules are presently known. The way out of this difficulty is to use the fact that a simple power transformation will transform the exponential distribution into a Weibull distribution.

But which Weibull distribution do we want? Clearly we want one that is as much like a normal distribution as possible. But this is too vague a requirement. A particularly desirable characteristic is that the mean and the median should be the same. This would permit the supplementary test for runs on either side of the center line to be used. It happens that a Weibull distribution with a shape parameter of 3.6 has a skewness of zero and a kurtosis of 2.72. This compares to a normal distribution which has a skewness of zero and a kurtosis of 3. A probability-paper plot of these two distributions is given in Nelson (1976). From this it can be seen that the two distributions are very closely matched from $P = 0.01$ to $P = 0.99$, and quite adequately matched out to $P = 0.001$ and $P = 0.999$.

Therefore, the required transformation is simply to raise the inter-event count to the 1/3.6 power. When this is done, all of the supplementary tests for special causes can be applied to the resulting individuals chart.

The procedure for setting up the control chart follows.

1. Examine each item.

2. Wait until the first nonconforming item is found. Then start to count the number of items produced until the next nonconforming item appears. Call this value X_1. Now start the counting over again and count the number of items produced until the next nonconforming item appears. Call this value X_2. Repeat until a preliminary sample of at least two dozen X values has been obtained.

3. Apply the power transformation to obtain values of $Y_i = X_i^{1/3.6} = X_i^{0.2777}$.

4. Set up the individuals chart using the Y values as follows. Calculate the mean of the Y values ($\bar{Y}$) and the mean of the moving ranges of 2 ($\overline{\text{MR}}_2$). The chart will have a centerline of $\bar{Y}$ and lower and upper control limits of $\bar{Y}-2.66\overline{\text{MR}}_2$ and $\bar{Y}+2.66\overline{\text{MR}}_2$, respectively.

5. Continue to count X values and calculate and plot the corresponding Y values. Use the recommended supplementary tests for special causes (see Nelson (1984)).

In contrast to the individuals chart discussed here, it is possible to develop a chart based on counts of n conforming items until a total of m (with m greater than 1) nonconforming items have been found. This can be modeled with a negative binomial distribution (see Craig (1953)). Note that, contrary to p charts, points below the lower control limit indicate deterioration of quality whereas points above the upper control limit show improved quality.

References

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