Control Chart Techniques For Time Oriented Rare Event Data

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ABSTRACT

The use of control chart techniques for time oriented rare event data is problematical. As the number of events being controlled diminishes, standard control chart techniques such as Individual Value / Moving Range, lose their effectiveness due to the small amount of data. Plotted points become a series of zeros with sparse occurrences of--usually--single events. Additionally, with rare event data such as warranty returns where the area of opportunity is not unique for each event, the conditions for control charting Poisson data (total count of nonconformities) or binomial data (number of units with nonconformities) is violated. This paper will explore the use of 'cycles until an event data' and compare the use of geometric control charts to Individual Value / Moving Range charts.

INTRODUCTION AND THEORY

What is a rare event? When the occurrence of a defect or problem (event) is better measured in parts per million than percent, then the use of classic control charts becomes difficult.

Normally, an investigator will collect and record the occurrence of an event (be it a defect, a warranty return, or a measurement value) using logical sub grouping and control charting techniques such at the Xbar/R or p-charts. When logical subgroups are not apparent, the data is collected in a time order and the Individual/Moving Range (I/mR) chart is used.

However, as data becomes 'rare' the required size of a logical subgroups becomes extremely large. For instance, warranty data is often collected and evaluated using a subgroup size of one year, and progress is evaluated by comparing year to year data. Because of the long time involved in collecting data that requires large sub group sizes, the immediate determination of common cause versus special cause, or within versus between variation is difficult.

This problem led to various investigators such as Benneyan [1,5,10] and Zhang [2] to investigate control charting techniques that calculate and plot the number of cycles occurring until (or between) events. It has also been shown that this type of data (cycles until an event) follows a geometric distribution, which is a special case of the negative binomial distribution as shown in Figure [Figure 1]

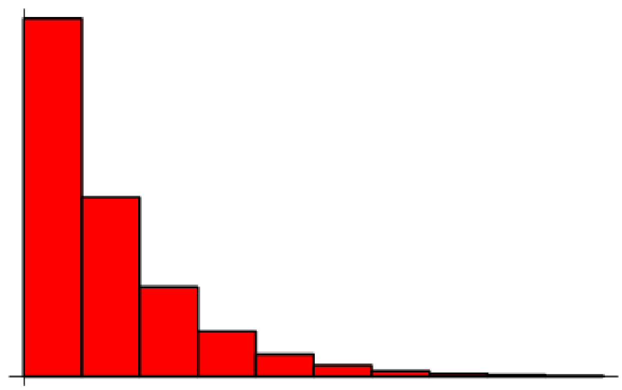

Figure 1 - Geometric Distribution (Frequency vs. Response)

This article explores control charting rare event data using 'cycles until event' data, various methods of calculating control chart limits, and applies these techniques to examples from industry as well as from our daily life. These control charts are called geometric control charts or "g-charts."

DESCRIPTION

EVENTS AND CYCLES

An 'event' is any measurable or notable occurrence of a defect or problem. Benneyan [1] explored post surgical infections and occurrences of accidental needle sticks. In industry, warranty returns, customer complaints, or final test results are examples of event data that may fall into the rare event category.

A 'cycle' is a general term for any time structured area of opportunity. Cycles are often expressed as 'days until an event' or 'days between events.' Cycles might also be number of units processed until an event or number of procedures performed until an event.

TYPES OF DATA

Though other categories may exist, for purposes of this paper data is usually divided into two types—aggregate data and specific data.

Aggregate data--in which many types of events are counted as defects such as;

- a final test station in an assembly process. Final test often is looking for a wide range of problems that could include performance issues, appearance issues, and labeling issues.
- Warranty data in which the measured event might be anything related to form, fit, or function.

Specific data--in which the defect is a particular item. For instance;

- measurement of a feature (diameter, resistance, color spectrum)
- measurement of a specific defect (voids in solder joints, label errors, presence of cracks in item).

AREAS OF OPPORTUNITY

The population from which rare event data is culled is usually referred to as an Area of Opportunity. This data may be collected from overlapping areas of opportunity or unique areas of opportunity.

- Overlapping Areas of Opportunity – defects found during the warranty period have overlapping areas of opportunity. New units--be they radios, entire cars, or video games--are constantly entering the population and old units are constantly be removed from the population. See [Figure 2].

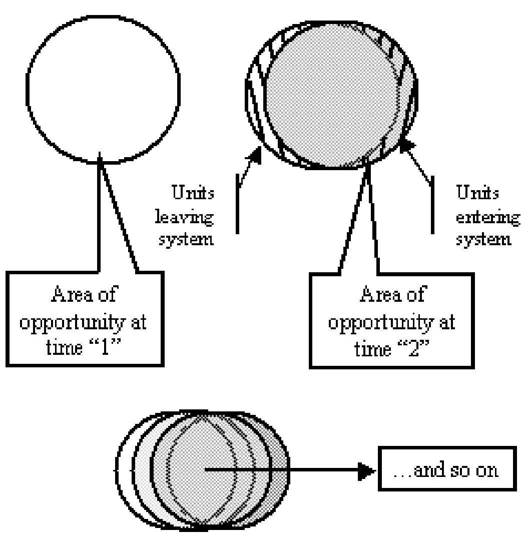

Figure 2 - Overlapping Areas of Opportunity

- Unique Areas of Opportunity – samples of data taken from progressively built products often fit into this category. For instance, if an event (defect) occurs after 100 units have passed through the station, those units are now gone and no longer part of the area of opportunity. The next area of opportunity is fluid and will be set once the next event occurs. See [Figure 3].

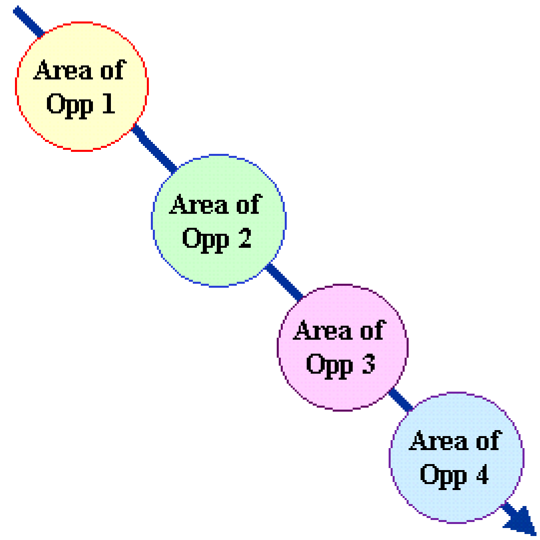

Figure 3 - Distinct Areas of Opportunity

In selecting the appropriate cycle measurement, it is most desirable to select an area of opportunity that is unique over areas of opportunity that are overlapping, i.e. if possible chose the 'number of units until an event' instead of the 'number of hours or days until an event.'

In selecting the type of 'event' data to chart, the usual questions apply; what is the desired outcome of the plotted data? What does the investigator wish to control?

The chart shown in [Figure 4] is the numbers of days until a specific type of customer complaint, in this case design issues for a particular product classification. An 'event' is recorded when the customer determines that a product has a defect. The cycle in this case is days. Therefore the cycles until event data for this case is days until a customer complaint.

Because the supplier sends new products into the customer's inventory each day, and because the customer assembles these parts into cars each day, the area of opportunity is best described as overlapping [Figure 2] .

[Figure 5] shows the data plotted in a histogram and confirms that Days Until Customer Complaint indeed fits into a geometric distribution.

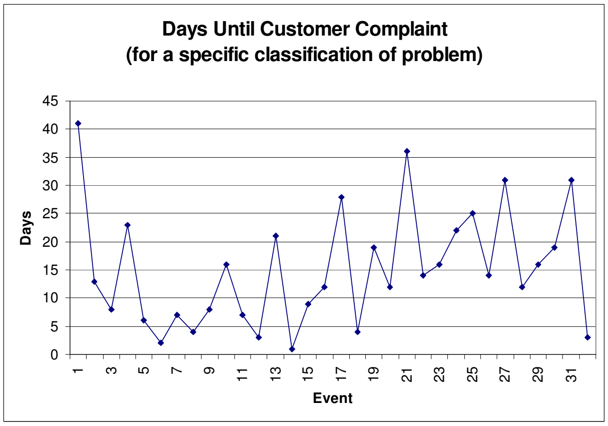

Figure 4 - Days Until Customer Complaint – Design Issues

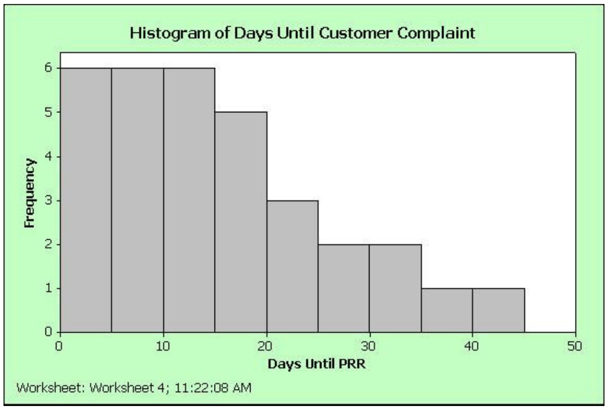

Figure 5 - Distribution of Days Until Customer Complaint – Design Issues

This data was collected over two years time. The number if these types of events were seventeen in the first year and sixteen in the second year. However, the "Days Until" g-chart shows that there was a potential dip in the data during the third quarter of the first year (points 5 through 9) and since that time, the "Days Until an Event" have an apparent rising trend. It is admitted that trend analysis on geometric control charts is an area that is not covered in this paper and requires more study.

THE GEOMETRIC DISTRIBUTION

The Distribution Parameter

The geometric distribution is shown in [Figure 1] and described in Zhang [2] as shown in Eqn (1)

$$P(X = x) = p(1-p)^x \quad ; \tag{1}$$

where "x" is the number of the event and "p" is the overall probability that an event will occur. This probability may be estimated from the data set thusly;

p = total events recorded / total cycles (2)

From the probability parameter, the expected value, E(X), shown in Eqn (3) and Variance V(X), shown in Eqn (4) may be derived.

$$E(X) = ((1-p)/p) \tag{3}$$

$$V(X) = (1-p)/p^2 \tag{4}$$

It is an interesting property of the geometric distribution that if one calculates the square root of the variance to determine an estimate of standard deviation, that this number is often very close to the expected value (or mean) of the data.

Control Limits Calculations for g-charts

Control limits for g-charts is a debated subject in the literature. Yang et. al. [9] references calculating the confidence intervals from the probability function as shown in Eqns (5,6). For this study an $\alpha/2$ of 0.025 was used.

$$UCL = LN(\alpha/2) / LN(1-p) - 1 \tag{5}$$

$$LCL = LN(1-\alpha/2) / LN(1-p) \tag{6}$$

Both Yang [0] and Zhang [2] suggests potential improvements to these limits but those are beyond the scope of this paper.

Benneyan [1,6] chose to multiply the estimate of the standard deviation by three and add or subtract this value from the mean of the data. Therefore, the Control Limits (CL) using this method may be expressed as shown in Eqn (7).

$$CL = Mean \pm k\sqrt{\frac{(1-p)}{p^2}} \tag{7}$$

Where k = number of "standard deviations" on each side of the center line desired for the chart, usually k=3. However, since the estimate of standard deviation is approximately equal to the mean, the lower control limit is most often zero or one (depending on whether 'events between' or 'events until' data is counted).

Because of the nature of the geometric distribution, the lower control limit is quite often at or near a value of zero or one. Therefore it is difficult for the investigator to know when the process is degrading significantly. Benneyan [10] suggests using Eqn (8) to determine the number of successive occurrences of an event at the lower control limit, which would indicate that the process has significantly degraded.

Number Events in Succession = LN(α)/LN(p) (8)

In this equation, α is the false alarm probability.

For many scenarios when this formula is used, two successive events at the lower control limit warrant investigation as a special cause event.

An alternate method of calculating control limits, and one investigated as a possibility in this paper, is to use the standard I/mR methods of calculating upper and lower control limits. This method, while less statistically robust, offers the advantage of simplicity.

The differences in these three values is exhibited for the "Days Until Customer Complaint–Design" data as charted in [Figure 4] and calculated in [Table 1].

Customer Complaints (Days Until)	
Mean	15
95% Confidence Interval (U)	53
95% Confidence Interval (L)	1
X/mR UCL	46
X/mR LCL	1
g-chart UCL	59
g-chart LCL	1

Table 1 - Comparison of Control Chart Limits

We can now plot these values on our g-chart and look for data points that are outside or near these limits.

SAMPLE DATA AND EXAMPLES

EXAMPLE ONE – US AIR ACCIDENTS

Air accident data was downloaded from NASDAC [7] and NTSB [8] web sites.

This example looks at the number of fatal air accidents in the United States between 1990 and 2005. A plot of this data as a year to year summary is illustrated in [Figure 6].

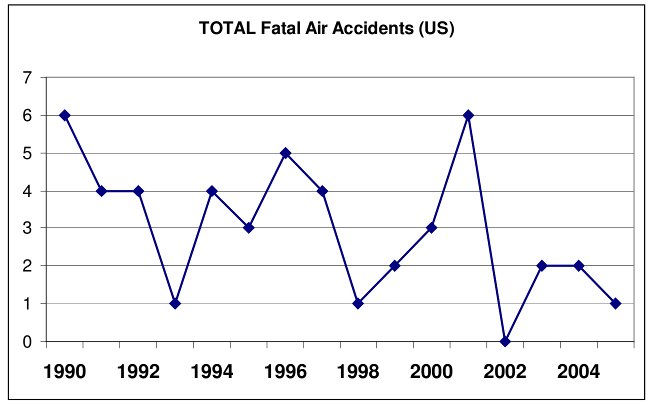

Figure 6 - Total Fatal Air Accidents by Year

However, the data also includes the date that each accident occurred and may therefore be turned into "Cycles Until" data and plotted using the geometric (g) control chart method. This allows for the investigator to begin to study the trends and occurrences of accidents in more detail. The g-chart for Days Until Fatal Air Accident is shown in [Figure 7] and the histogram confirming the geometric distribution of the data is in [Figure 8].

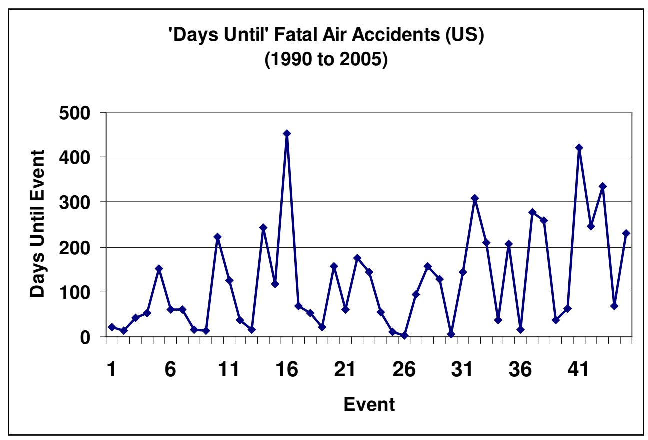

Figure 7 - Days Until Fatal Air Accident (US)

Figure 8 - Histogram of Days Until Air Accident

It may be noted that the 'Days Until' data [Figure 7] seems to agree with the trend seen in [Figure 6], and that tragic air accidents are indeed decreasing over time, or conversely, that the 'Days Until' a tragic air accident are increasing.

However, we can now select our control limits and begin to see if any special cause events are evident and if our "process" is in control. [Figure 9] illustrates the same data but includes the additional statistics.

The Individual chart reveals that the data at point sixteen is 454 days until a fatal air accident. This value is over the I/mR UCL and very close to the upper probability limit. See [Table 2] for the exact control limits.

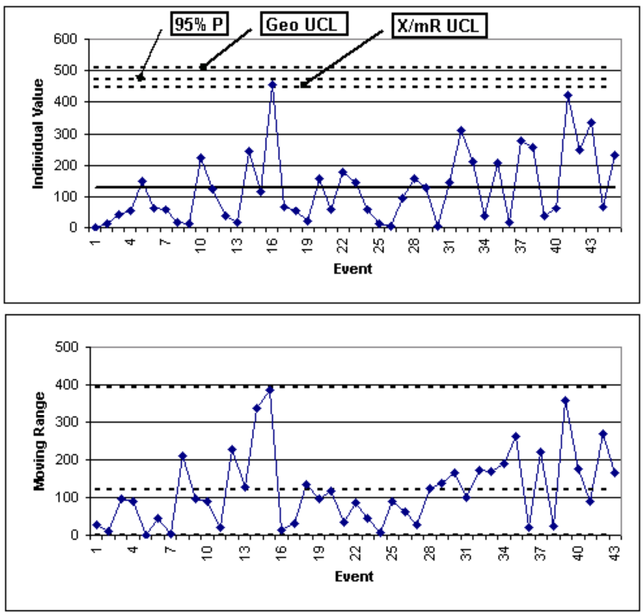

Figure 9 - I/MR Chart Including Alternate UCLs

Finally, the data at point twenty-six is "3" days until an event. This value equals the lower control limit as calculated by the probability function, and therefore may be a significant deviation from a standard geometric distribution.

The number of events in succession calculated per Eqn (8) is less than one, indicating that any two points in a row at the lower control limit would be significant.

However, there are no cases of this condition in this data set, so it appears that no special cause is occurring at the lower control limit.

Fatal Air Accident Data (Days Until)	
Mean	128
95% Confidence Interval (U)	468
95% Confidence Interval (L)	3
X/mR UCL	447
X/mR LCL	1
Geometric Mean UCL (Benneyan [1]	509
Geometric Mean LCL (Benneyan [1]	1

Table 2 - Upper and Lower Control Limits by Method

As can be seen, the g-chart is very good at highlighting what went "right" and weaker at determining points where something went wrong. It is not certain that the I/mR "Range" chart adds any information of value since the data in the range chart tends to mirror the data in the Individual chart and 'stability' with respect to a geometric distribution does not apply to stability when judged against a normal distribution.

EXAMPLE TWO – CUSTOMER COMPLAINT DATA REWORK

Customer complaint data to a supplier was sorted by those complaints that involved customer perceived rework issues.

This data is being tracked in order to monitor and gage improvements to the manufacturing process and their effectiveness with respect to the customer.

[Figure 10] shows this data in I/mR format with the three methods of calculating control limits plotted. [Table 3] shows the actual values of these limits.

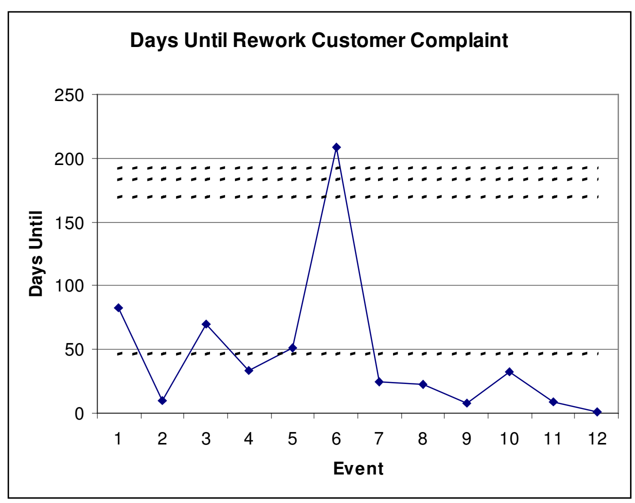

Figure 10 - Days Until Customer Complaint – Rework

Customer Complaints - Rework (Days Until)	
Mean	46

95% Confidence Interval (U)	167
95% Confidence Interval (L)	1

X/mR UCL	192
X/mR LCL	1

g-chart UCL	183
g-chart LCL	1

Table 3 - Customer Complaints - Rework

For this data the trend is apparently moving toward fewer days until a customer complaint, even though one "good" out-of-control event occurred at point six. In real terms this means that customer complaints for this type of condition did not occur for over 200 days, and then the number of days until an event decreased to a mean of only about sixteen. Of the many reasons for this decrease the following are common;

- new products being introduced to new lines, thereby causing a learning curve in the number and quality of reworked product
- a general reduction in the quality of rework for the products included in this data
- Increased attention by the customer to these types of problems, which is reflected in more items categorized as official customer complaints than in the past.

EXAMPLE THREE – ELECTRICAL TEST STATION

Lastly, an electrical test station is operating with a first pass yield of about 98%. Data was collected for an eighteen hour period and sorted by one specific model of product. The total number of units tested were 2216 boards.

The Individual and Moving Range charts of the statistic "Units Until a Reject" are shown in [Figure 11] plotted with all three of the alternate control limits (I/mR, confidence interval method, and the three times standard deviation method).

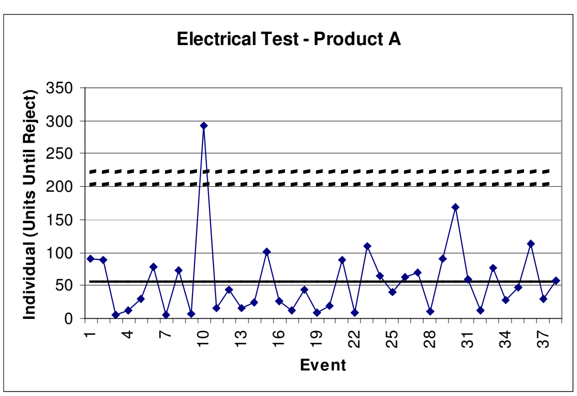

Figure 11 - I/mR of Units Until Electrical Reject

No trend is seen over this time period, but one interesting 'good' point may be observed that should be investigated for special cause to determine 'what went right.'

An hourly First Time Yield chart is shown in [Figure 12]. This chart highlights the differences between controlling 'Cycles Until Event' data versus time oriented yield data. The 'Units Until' chart of [Figure 11] highlights the long period of time that occurred without a reject (between 4:00 AM and 6:00 AM), while the Yield chart [Figure 12], highlights the low yield (96%) during the seven o'clock AM hour. Calculating the I/mR lower control limit of this data results in a LCL of 94.5%, therefore this point does not appear statistically significant based on this limited data set and we are uncertain whether to investigate 'what went wrong' at this data point.

That same information can be found in Figure 11 - I/mR of Units Until Electrical Reject, and corresponds to data points sixteen through twenty. These points do seem to be grouped below the mean, but in no case does more than one point hit the lower control limit in succession.

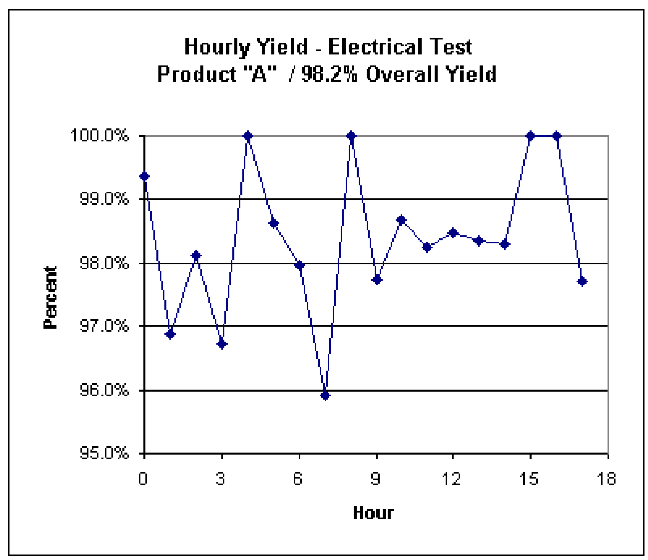

Figure 12 - Electrical Test Yield by Hour

Finally, the hourly yield data indicates four points that are at 100% yield. However, because upper control limits for yield data become unreliable as yield increases (they often calculate to values over 100%), the investigator cannot be sure if those points are significant. Deeper investigation shows that during hours fifteen and sixteen, the total number of units run was very low. Therefore, 100% yield values during these hours do not mean much to the investigator.

However, in Figure 11 - I/mR of Units Until Electrical Reject, we are directed to concentrate on the time period around point 10, where we went for almost 300 units before observing another reject.

CONCLUSIONS

Cycles until an event data is an intriguing alternate tool available to an investigator who is dealing with rare event data. This paper does not purport to answer all the questions related to control charting of 'cycles until an event' data but does attempt to open the subject for peer debate.

The following observations were made from the various data sets analyzed in this study.

- Cycles Until Event control charting is an effective way to evaluate trends and patterns in rare event data.

- When collecting 'cycles until a rare event' data, be aware of the type of area of opportunity (overlapping or unique) and the type of data (aggregate or specific). When possible select *specific* data from a *unique* areas of opportunity

- Upper control limits give the investigator an indication of where to investigate 'what went right' in the process.

- Lower control limits indicate points to investigate 'what went wrong' but are quite often near a value of one due to the nature of the geometric control chart. However, when more than one event occurs in succession at the lower control limit, this should be taken as a possible indication of special cause.

- Even though not demonstrated in this paper, the power of any control chart is in establishing control limits on a stable process and using these limits to gage future events.

- The 95% confidence intervals calculated from the probability function (α = .05, two tailed distribution) seem to be a good choice of calculating control limits. However, one will most likely make the same decisions regardless of which method of control limit calculations are used.

- The Individual/Moving Range control chart offers advantages for plotting 'cycles until event' data. Despite their lower level of statistical robustness, one can gain a quick indication of any exceptionally long cycles until an event. Also one can use the I/mR chart to get a quick look at any repeating events during the same cycle (repeating values of "one" on the chart). This is helpful because, many statistical calculation packages include automatic I/mR chart creation, but do not include g-chart options.

- When investigating high yield production lines that are generating units with unique areas of opportunity (e.g. a flow line), record and plot the number of units until a defect rather than time until a defect. This helps the investigator to remove the effect of production downtimes and weakness of behind using time as a logical subgroup.

- In retrospect, it is no surprise that in high yield processes, defects quite often present themselves in closely associated groups—with only one or a few cycles between each defect. It is the nature of the geometric distribution that low 'cycles until an event' occur more often than high 'cycles until an event.'

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